Synchronization of mutually coupled chaotic systems

D. Y. Tang and N. R. Heckenberg
Physics Department, The University of Queensland, Brisbane, Qld 4072, Australia
(Received 3 December 1996)

We report on the experimental observation of both basic frequency locking synchronization and chaos synchronization between two mutually coupled chaotic subsystems. We show that these two kinds of synchronization are two stages of interaction between coupled chaotic systems. In particular the chaos synchronization could be understood as a state of phase locking between coupled chaotic oscillations. [S1063-651X(97)03806-3]

PACS number(s): 05.45.+b, 05.40.+j, 42.65.Sf, 42.60.Mi

I. INTRODUCTION

Synchronization between periodic oscillations of mutually coupled dynamical systems is a well-known phenomenon. Generally, when the oscillation frequencies of two coupled periodic systems are within a certain range called the locking range, the frequencies will automatically lock to a mutual value and consequently both systems oscillate with the same frequency. After frequency locking between their oscillations, we say they are synchronized. Since the oscillation of a periodic system is regular, the effect of synchronization between them is clear and unique.

The dynamics of a system can also be chaotic. Recently, there has been great interest in synchronization between chaotic systems [1–11]. In contrast to the oscillation of a periodic system, the oscillation of a chaotic system is dynamically intrinsically unstable: Its oscillation depends sensitively on the initial conditions and varies with time. Due to this special character of chaotic systems, there have developed different versions of the definition of synchronization between chaotic oscillations. Mostly, synchronization of chaotic oscillations is defined as the complete coincidence of the trajectories of the coupled individual chaotic systems (subsystems) in the phase space [5]. According to this definition, under the synchronization the dynamics of two coupled systems (subsystems) become exactly the same, even though without coupling they are not dynamically identical. This kind of synchronization was called ''chaos synchronization'' and has been observed in coupled chaotic systems [2–10].

Another definition takes account of the behavior of some chaotic attractors that in their power spectrum a basic frequency can be distinguished and defines synchronization of chaotic oscillations as meaning merely that their basic frequencies are locked together [1]. We refer to this synchronization as the ''basic frequency locking synchronization.'' A chaotic attractor whose power spectrum possesses this behavior is called a ''phase coherent attractor'' [12–14]. A major property of these chaotic attractors is that their chaotic behavior results mainly from chaotic amplitude modulation and the contribution from the chaotic phase modulation is very weak. Consequently, there exists a predominant frequency in the chaotic oscillation of these attractors. A characteristic of this synchronization is that the average oscillation frequencies of the coupled chaotic systems are entrained, while the amplitudes of the oscillations remain chaotic and independent. An advantage of this definition is that, like the synchronization between periodic systems, the mechanism of synchronization is clear. Rosenblum, Pikovsky, and Kurths have reported an observation of ''phase'' synchronization between coupled chaotic systems [15]. However, the synchronization they referred to seems to be exactly the basic frequency locking synchronization. Strictly speaking, despite the fact that the phase fluctuation of the oscillation of a phase coherent strange attractor is very small, under the basic frequency locking, the instantaneous phases of these coupled systems are not locked.

In this paper we report on an experimental observation of both basic frequency locking synchronization and chaos synchronization between two mutually coupled chaotic subsystems. We show that, like coupled periodic systems, coupled chaotic systems have a tendency to engage in mutual synchronization in the form of basic frequency locking or chaotic phase locking. Our experimental results demonstrate that the two observed synchronizations are in fact the two natural stages of interaction between coupled chaotic systems. In particular the chaos synchronization between chaotic systems could be physically understood as a result of phase locking between coupled chaotic oscillations.

II. EXPERIMENT AND RESULTS

Our experimental system is an optically pumped NH_3 bidirectional ring laser. Details of the configuration of the laser were reported in [16]. This laser was chosen for the present experimental study because it lases in two modes simultaneously and these two modes are mutually coupled. One mode field of the laser propagates in the same direction as the pump laser beam and is called the forward mode; the other mode field propagates against the direction of the pump laser beam and is called the backward mode. Due to the optical pumping of the laser that selectively excites NH_3 molecules with the same longitudinal velocity, the gain bandwidth of the laser is very narrow, limited by homogeneous broadening. Both modes of the laser share the same population inversion, while, because of the Doppler effect resulting from the motion of the excited molecules, the effective gain frequency of each mode is different. The frequency difference between them is determined by the pump frequency detuning relative to the NH_3 absorption line center. This relation between the two laser modes results in a

1063-651X/97/55(6)/6618(6)/$10.00

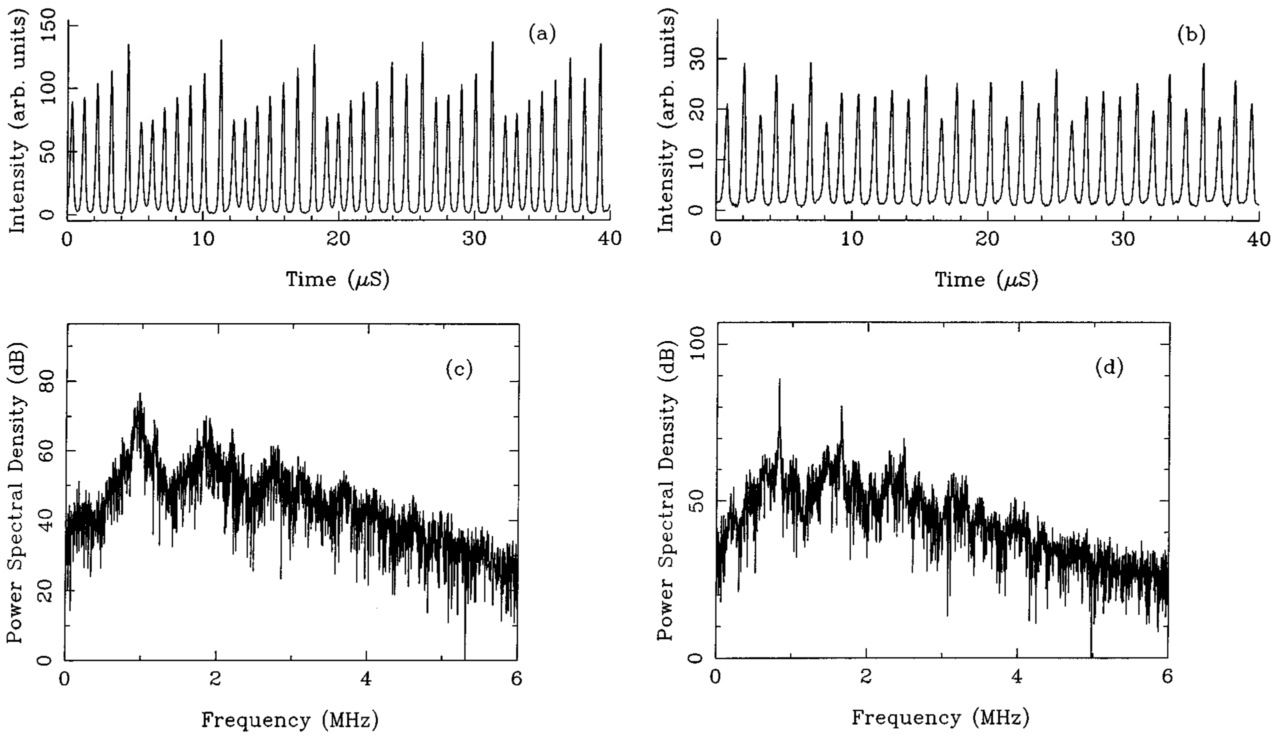

FIG. 1. Typical chaotic dynamics of the modes without coupling: (a) mode intensity evolution of a Lorenz-like spiral chaos, (b) mode intensity evolution of a period-doubling chaos, (c) Fourier power spectrum calculated from data shown in (a), and (d) Fourier power spectrum calculated from data shown in (b).

strong cross saturation between their gains. Another coupling mechanism between the two modes is the backscattering resulting from the dynamical spatial population inversion grating formed by the two counterpropagating mode fields, and this coupling causes further a strong phase-dependent interaction between them. Because these two mode fields propagate in opposite directions, they separate on the out-coupling mirror of the laser, which allows the dynamics of each coupled mode to be easily detected separately, even though they are mutually coupled in the laser cavity.

Depending on the pump laser frequency setting, this laser can also operate single mode in either of these two modes. It was found previously that under suitable conditions, the single-mode operation of the laser can exhibit different kinds of deterministic chaos, such as Lorenz-like spiral chaos [17], period-doubling chaos [18], and type-III intermittent chaos [19]. The single-mode chaotic dynamics was found to be an intrinsic behavior of the laser and, as we will show below, in the parameter range producing single-mode chaos, when the laser operates in the multimode emission, each mode can exhibit chaotic dynamics as well. As examples of the single-mode chaos of the laser, we show in Fig. 1 a typical Lorenz-like spiral chaos and a period-doubling chaos observed in the laser together with their Fourier power spectra. Studies of the dynamics of these forms of single-mode laser chaos have revealed the following behaviors. First, as can be seen in the spectra shown, there exist distinguishable sharp spikes in the power spectrum of the single-mode laser chaos. These spikes are superposed on the broadband background of the spectrum. Experimentally, it was further found that these spikes are more significant at the onset of each kind of chaos and as the chaos of the mode increased, their width increases and finally they disappear. This behavior of the single-mode laser chaos shows that when the mode dynamics is not very chaotic, its chaotic attractor is indeed a phase coherent attractor. Second, the position of the fundamental spike in the spectrum depends measurably on the concrete laser conditions such as the gain and the laser cavity detuning. This indicates that the basic frequency of the chaotic mode intensity oscillation changes with the mode conditions. Third, because of the longitudinal optical pumping of the laser that breaks the symmetry between the forward- and the backward-mode emission of the laser, even with zero pump laser frequency detuning relative to the NH_3 gas absorption line center, the gains for the forward and the backward emissions are not the same. As demonstrated by Heppner *et al.* [20], in the steady-state operation of the laser, the ac Stark splitting in the forward gain is more significant than in the backward gain. This asymmetry between the two modes leads to a significant difference in their detailed chaotic dynamics. When the pump laser frequency is detuned from the NH_3 gas absorption line center, a further asymmetry in the laser conditions between these two modes results.

In the present experiment, we are interested in the synchronization between the chaotic dynamics of two mutually coupled chaotic systems. Regardless of the concrete coupling mechanism between the two modes of our laser, one can in principle view each mode of the laser as a subsystem and regard the dynamics of the whole laser as results of the mode interaction. Because in the case of our laser we can obtain and measure the chaotic dynamics of each individual mode with and without coupling simply by changing pump laser frequency detuning relative to the NH_3 gas absorption line center, this treatment is also a practical way of understanding

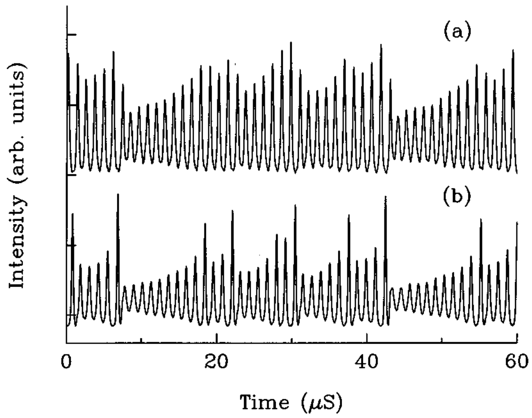

FIG. 2. Mode intensity evolution of each of the two coupled modes under basic frequency locking synchronization: (a) intensity evolution of the forward mode and (b) intensity evolution of the backward mode. The pump intensity is 2.4 W/cm^2, the NH_3 gas pressure is 5 Pa, and the output mirror mesh constant is 51 μm.

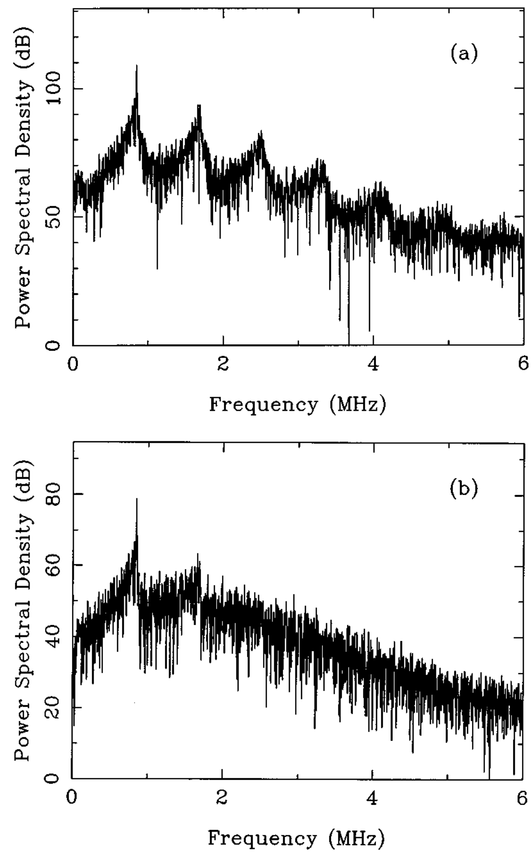

FIG. 3. Fourier power spectra calculated from the data shown in Fig. 2: (a) corresponding to the forward-mode intensity evolution of Fig. 2(a) and (b) corresponding to the backward-mode intensity evolution of Fig. 2(b).

the complex dynamics of the laser.

Experimentally, we used a separate Schottky-barrier diode to simultaneously detect the intensity evolution of each of the laser modes and studied their dynamics under different conditions, e.g., different pump intensity, gas pressure, and cavity loss. Since in our experiment with a properly selected cavity detuning changing the pump frequency detuning is effectively equivalent to changing the coupling strength between the two modes, the pump frequency detuning relative to the NH_3 gas absorption line center has been chosen as the control parameter.

Generally, it is observed that when the chaotic modes are coupled, the intensity evolution of each mode becomes very complicated. The chaotic dynamics of each mode under coupling is very different from that of the single-mode chaotic dynamics of the laser; particularly, normally no obvious synchronization between the modes was observed. This behavior of the coupled chaotic modes is presumably a reflection of the high-dimensional character of the system. However, we find that under certain conditions, even though the coupling strength between the modes is not strong, the intensity dynamics of the coupled modes can spontaneously become synchronized. Two kinds of synchronizations between the chaotic mode intensity dynamics were observed in our experiment. As an example, one of these synchronizations is shown in Fig. 2, which can be classified as the basic frequency locking synchronization. From Fig. 2 it is clear that the intensity evolution of both modes is chaotic. Analyzing the intensity evolution of each mode, it was further found that their chaotic dynamics mainly retains the characteristics of the single-mode chaotic dynamics of the laser. Because the conditions for each mode are not all the same, their exact chaotic mode intensity evolutions are different. But the chaotic intensity pulsation rate of each mode is not independent, despite of the fact that the dynamics of each mode is chaotic and their detailed evolutions are different. Another significant feature of the mode intensity variation shown is that the individual intensity pulsations of the two modes are always out of step. Under different conditions, a kind of in-step chaotic mode intensity pulsation relation between the two modes was also observed [21]. The out-of-step pulsation of coupled periodic systems has been observed and intensively investigated before [22–24]. It was identified as a cooperative self-organization of coupled systems. In contrast to the case of coupled periodic systems, our experimental results demonstrate that even in the case of coupled chaotic systems, under the interaction between them their chaotic dynamics can be cooperatively self-organized.

To further show that the basic oscillation frequency of the chaotic intensity evolutions shown in Fig. 2 are frequency locked, we have calculated their intensity power spectra and shown them in Fig. 3. It is easy to identify the fundamental sharp spike and its harmonics in the power spectra shown in Fig. 3. These sharp spikes superposed on the broadband background and the position of the fundamental spike gives the basic mode chaotic intensity pulsation frequency. As expected, the positions of the fundamental spikes of these two modes are exactly the same, indicating that they are actually average frequency locked. Apart from the positions of the line spikes in the spectrum, other structures of the two power spectra are totally different. This shows again from a different aspect, that under the basic frequency locking, the detailed dynamical behavior of the synchronized chaotic systems could be very different. We note that generally under the interaction between the modes, the phase coherence of the uncoupled attractors is destroyed. Only under the synchronization is this behavior of the attractors retained.

Although under this basic frequency locking synchroniza-

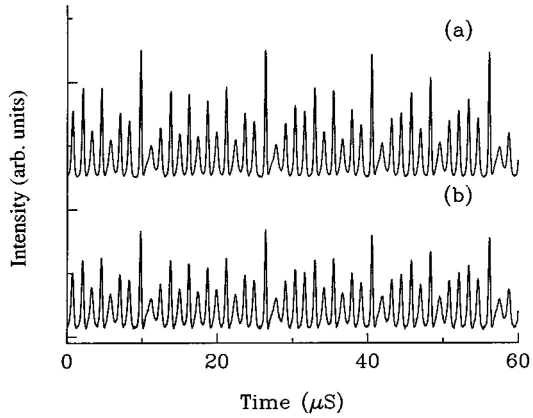

FIG. 4. Mode intensity evolution of each of the two coupled modes under chaos synchronization: (a) intensity evolution of the forward mode and (b) intensity evolution of the backward mode. The pump intensity is 3.5 W/cm^2, the NH_3 gas pressure is 3.5 Pa, and the output mirror mesh constant is 102 μm.

tion the detailed chaotic mode intensity evolutions of each mode are different, the envelopes of these two mode intensity pulsations do show some similarities, as could be identified in Fig. 2. This rough similarity shows that apart from the basic frequency locking between their chaotic oscillations, there exists also a tendency of approaching chaos synchronization between their chaotic dynamics. Based on this experimental result, we postulate that the basic frequency locking could be a primary result of the interaction between coupled chaotic systems, and its realization requires fewer conditions. When more conditions are fulfilled, further ''synchronization'' between the chaotic intensity dynamics of the coupled modes could be achieved. To this end we have experimentally investigated the possibility of chaos synchronization between the chaotic intensity dynamics of the modes under a wide parameter range. We find actually that under increased cavity loss, this kind of chaos synchronization can be observed in the laser as shown in Fig. 4. Under this synchronization, the chaotic intensity variations of the two modes are always exactly identical, even though, as mentioned above, in our laser the dynamical behaviors of the two modes are not normally identical.

III. RELATION BETWEEN THE TWO SYNCHRONIZATIONS

From our experimental results, it seems that both the basic frequency locking synchronization and the chaos synchronization between coupled chaotic systems are two natural results of chaotic interaction between them. To better understand the interaction between coupled chaotic systems and especially to find out the relationship between these two kinds of synchronization, we examine below the behavior of these two synchronized states. In studying the behavior of a period-doubling chaos chaotic signal, Farmer and his coworkers noticed that a chaotic evolution could be considered as consisting of two parts—the chaotic amplitude modulation and chaotic phase modulation—and introduced the idea of understanding the behavior of a chaotic system in analogy with that of an oscillator [12,13]. Based on Gabor's phase definition for an arbitrary signal [25], Rosenblum, Pikovsky, and Kurths have defined the phase of a chaotic signal [15]. Following Gabor's definition, the analytic signal $\psi(t)$ is a complex function of time defined as

$$\psi(t)=S(t)+i\widetilde{S}(t)=A(t)e^{i\phi(t)}, \tag{1}$$

where $S(t)$ is a real function of time and the function $\widetilde{S}(t)$ is the Hilbert transform of $S(t)$,

$$\widetilde{S}(t)=\frac{1}{\pi}\,\mathrm{P}\int_{-\infty}^{\infty}\frac{S(\tau)}{t-\tau}\,d\tau, \tag{2}$$

where P means that the integral is taken in the sense of the Cauchy principal value. Equation (1) uniquely defines the instantaneous amplitude $A(t)$ and phase $\phi(t)$ of an arbitrary signal $s(t)$. Although from this definition, if an arbitrary chaotic signal $s(t)$ is known, by using the Hilbert transformation (2), one can always work out its instantaneous amplitude $A(t)$ and instantaneous phase $\phi(t)$; however, directly applying this definition to a completely chaotic signal is not very useful because the physical meaning of this calculated phase is unclear. As mentioned above, for phase coherent chaotic attractors, one can identify a basic frequency in their power spectrum. This frequency provides a unique reference frequency that can be used to define the phase of their chaotic variation. Therefore, for these chaotic oscillations one can write their instantaneous phase variation in the form

$$\phi(t)=\omega_0 t+\varphi(t), \tag{3}$$

where ω_0 is the basic frequency and $\varphi(t)$ can be defined as the chaotic phase modulation of these signals. The physical meaning of this defined chaotic phase is clear. While the basic frequency is the average pulsation frequency of a chaotic oscillation, the phase $\varphi(t)$ is then the instantaneous phase modulation on the phase evolution resulting from the basic frequency. This phase modulation is due to the chaotic behavior of a chaotic system, and because of this phase modulation the instantaneous frequency of a chaotic system varies with time.

With the help of the above definition of chaotic phase, we have calculated the associated chaotic phase evolution of each coupled chaotic system in the synchronized states. Figures 5 and 6 show these calculated results. Figure 5(a) shows the calculated phase evolutions of each of the coupled chaotic systems under the basic frequency locking synchronization shown in Fig. 2. Figure 6(a) shows the calculated phase evolutions of each of the two coupled chaotic systems under the chaos synchronization shown in Fig. 4. These phase evolutions show the common characteristic that there exists a big slope between the total phase evolution and the time axis, indicating that there exits a big average frequency in the phase evolutions. In both cases of synchronization the average frequency of each coupled systems is the same, showing that they are in an average frequency locked state. There exist also small phase modulations around this average phase slope. While in the case of chaos synchronization, the instantaneous small phase modulations of each coupled chaotic system are the same, the instantaneous small phase modulations of each coupled chaotic systems in the basic frequency

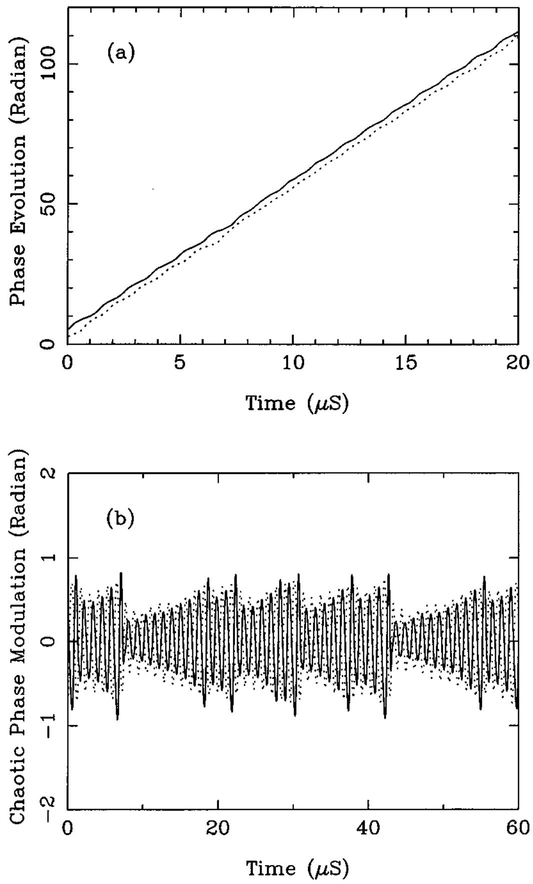

FIG. 5. Phase evolution associated with the chaotic dynamics of each of the coupled subsystems under basic frequency locking synchronization: (a) total phase evolution and (b) phase evolution after suppressing the phase change represented by the average slope. The solid line corresponds to the chaotic variation showing in Fig. 2(b) and the dotted line corresponds to the chaotic variations shown in Fig. 2(a).

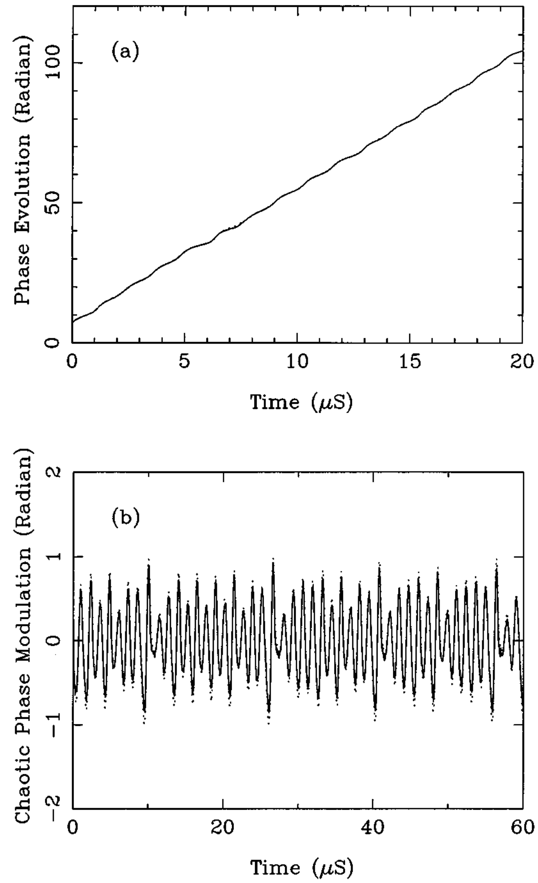

FIG. 6. Phase evolution associated with the chaotic dynamics of each of the coupled subsystems under chaos synchronization: (a) total phase evolution and (b) phase evolution after suppressing the phase change represented by the average slope. The solid line corresponds to the chaotic variation shown in Fig. 5(a) and the dotted line corresponds to the chaotic variations shown in Fig. 5(b).

locked case are clearly different. We checked that the average slope shown in the phase evolutions shown in Figs. 5(a) and 6(a) is the average oscillation frequency of each of the coupled chaotic systems.

Figures 5(b) and 6(b) show the phase evolution of each coupled system after subtraction of the phase change represented by the average phase slope. According to Eq. (3), this is the chaotic phase modulation of each of the coupled chaotic systems. This chaotic phase modulation of each system exhibits a clear similarity to the corresponding chaotic amplitude dynamics of the system. The instantaneous chaotic phase modulations of the two coupled systems shown in Fig. 2 show also an out-of-step phase variation. We see in Fig. 5(b) that although the chaotic phase modulation of each chaotic system is small, the difference between the instantaneous chaotic phase modulations of the systems is not constant, showing that they are not in a phase locked state. This experimental result negates the existence of the ''phase locked state'' supposed by Rosenblum, Pikovsky, and Kurths [15]. In contrast, Fig. 6(b) shows that within the experimental error range, the instantaneous chaotic phase modulations of the two coupled systems shown in Fig. 4 are the same, showing that they are actually in a chaotic phase locked state.

Comparing the phase variations shown in Figs. 5 and 6, the relationship between these two synchronized chaotic states becomes clear. The chaos synchronized state is in fact a chaotic phase locked state. We see that by regarding the behavior of a chaotic system as a chaotic oscillator, the interaction between two coupled chaotic systems could be well understood. As a universal behavior of coupled oscillators, when the basic oscillation frequency of coupled oscillators are within the locking range, their basic frequencies will lock together. A difference between the coupled periodic oscillator and the coupled chaotic oscillator is that in the case of the periodic oscillator, the phase of oscillations is intrinsically stable, so there is no difference between the frequency locking and phase locking of coupled periodic oscillators, while in the case of the chaotic oscillator, because the phase of the oscillations is intrinsically unstable, frequency locking is no longer equal to phase locking. In any case, we can see that the basic frequency locking and phase locking are two different stages of the same interaction between coupled chaotic systems.

IV. CONCLUSION

In conclusion, we have experimentally observed both basic frequency locking synchronization and chaos synchronization between the chaotic intensity dynamics of two mutually coupled laser modes. We found that the basic frequency locking synchronization between the chaotic intensity dynamics of the two coupled modes can be easily achieved. In comparison with the chaos synchronization case, it has a broad parameter range. Using the definition of phase of a chaotic signal proposed by Rosenblum, Pikovsky, and Kurths, we have calculated the chaotic phase evolution embedded in the chaotic dynamics of each coupled laser mode. Our experimental results show that the basic frequency locked chaotic state is not a chaotic phase locked state, but the chaos synchronized state is a chaotic phase locked state.

[1] V. S. Anishchenko, T. E. Vadivasova, D. E. Postnov, and M. A. Safonova, Int. J. Bifurcation Chaos **2**, 633 (1992).
[2] L. Pecora and T. L. Carroll, Phys. Rev. A **44**, 2374 (1991).
[3] R. Roy and K. S. Thornburg, Jr., Phys. Rev. Lett. **72**, 2009 (1994).
[4] L. Pecora and T. L. Carroll, Phys. Rev. Lett. **64**, 821 (1990).
[5] L. Kocarev and U. Parlitz, Phys. Rev. Lett. **76**, 1816 (1996).
[6] R. He and P. G. Vaidya, Phys. Rev. A **46**, 7383 (1992).
[7] A. A. Alexeyev and V. D. Shalfeev, Int. J. Bifurcation Chaos **5**, 551 (1995).
[8] C. W. Wu and L. O. Chu, Int. J. Bifurcation Chaos **4**, 979 (1994).
[9] T. Sugawara, M. Tachikawa, T. Tsnkamoto, and T. Shimizu, Phys. Rev. Lett. **72**, 3502 (1994).
[10] H. G. Winful and L. Rahman, Phys. Rev. Lett. **65**, 1575 (1990).
[11] K. M. Cuomo and A. V. Oppenheim, Phys. Rev. Lett. **71**, 65 (1993).
[12] J. D. Farmer, Phys. Rev. Lett. **47**, 179 (1981).
[13] J. Crutchfield, D. Farmer, N. Packard, R. Shaw, G. Jones, and R. J. Donnelly, Phys. Lett. **76A**, 1 (1980).
[14] E. F. Stone, Phys. Lett. A **163**, 367 (1992).
[15] M. G. Rosenblum, A. S. Pikovsky, and J. Kurths, Phys. Rev. Lett. **76**, 1804 (1996).
[16] Tin Win, M. Y. Li, J. T. Malos, N. R. Heckenberg, and C. O. Weiss, Opt. Commun. **103**, 479 (1993).
[17] C. O. Weiss and J. Brock, Phys. Rev. Lett. **57**, 2804 (1986).
[18] D. Y. Tang, M. Y. Li, J. T. Malos, N. R. Heckenberg, and C. O. Weiss, Phys. Rev. A **52**, 717 (1995).
[19] D. Y. Tang, J. Pujol, and C. O. Weiss, Phys. Rev. A **44**, R35 (1991).
[20] J. Heppner, C. O. Weiss, U. Hbner, and G. Schinn, IEEE J. Quantum Electron. **QE-16**, 392 (1980).
[21] D. Y. Tang and N. R. Heckenberg, Opt. Commun. **131**, 89 (1996).
[22] K. Wiesenfeld, C. Bracikowski, G. James, and R. Roy, Phys. Rev. Lett. **65**, 1749 (1990).
[23] I. B. Schwartz and K. Y. Tsang, Phys. Rev. Lett **73**, 2797 (1994).
[24] S. Nichols and K. Wiesenfeld, Phys. Rev. A **45**, 8430 (1992).
[25] D. Gabor, J. IEE London **93**, 429 (1946).